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Design and development of precision force transducers

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This paper presents design and development of precision ring shaped force transducers (FTs) based on theories of thin elastic rings for uniaxial forces in compression and tension mode (capacity 20 kN and 50 kN). A three dimensional model of FT has been developed and stress - strain and axial deflection patterns have been studied using software ABAQUS standard student edition 6.7.2. FTs are metrologically characterized according to calibration procedure based on ISO 376-2004 using 50 kN dead weight force machine with uncertainty of measurement ± 0.003% (k = 2). FTs are found to have uncertainty of measurement better than ± 0.025% (k = 2) including relative deviations due to repeatability error, reproducibility error, zero error, resolution error, interpolation error and uncertainty of measurement of force applied. FTs qualify class 00 per the standard ISO 376-2004 and may be used as force transfer standards.

Keywords: Finite element method, Force, Measurement, Strain, Stress

Introduction

Force transducers (FTs) have been playing a vital role in applications like testing of material testing machines, electronic weighing balance, weighing of aircrafts, thrust measurement of jet or rocket engines, monitoring components of cutting forces in different machining processes etc, thus insisting accurate and precise measurement of force. FTs are of various types (ring shaped, elliptical dynamometers type, strain gauged load cells or frequency based tuning fork type). Though ring shaped FTs have been used for many decades, a rationalized procedure for design and development is still awaited. Efforts have been made to design and develop circular ring shaped FTs with uniform strength[1] and to measure force exerted on a sphere by surrounding fluid[2]. It has also been attempted to develop expressions for stress - strain and axial deflection[3]. Efforts have been made to develop ring shaped three axis micro force sensor, but application was restricted to Smartpen™ only[4]. Suitable modifications suggested in the shape of circular ring like octagonal ring and extended octagonal ring FTs have provided limited applications to agriculture engineering[5], and monitoring various components of cutting forces in different machining processes. Developments were intended to some particular applications and uncertainty of measurement reported was high[6-10]. No efforts were made for development of precision ring shaped FTs, which could serve as a force transfer standard and uncertainty of measurement upto ± 0.025% ($k=2$). This study presents design and development of precision ring shaped FTs and their metrological characterization using calibration procedure based on ISO 376-2004.

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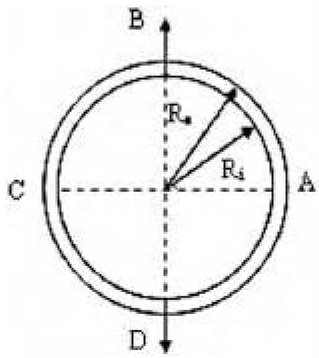
Experimental Section

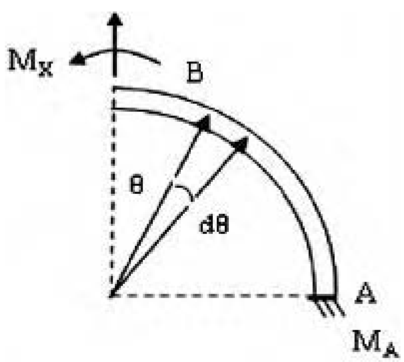
Analytical Study

Ring shaped FT has been considered as a circular ring that is symmetric to both axis (Fig. 1a) and one quarter of ring has been taken for analysis (Fig. 1b). Hence, ring is exposed to axial forces in either mode (force may be compressive or tensile). Free body diagram suggests one end of the quarter of ring is free, where force is applied and the other end is fixed (no rotation permitted) that moment M is statistically indeterminate and strain energy U for this quarter of ring is due to the bending moment M in the ring. From Castigliano's second theorem[1,2]

$$\partial U / \partial M_A = 0 \qquad \ldots(1)$$



a)



b)

Fig. 1 — Ring shaped force transducer (FT): a) Typical model; and b) Quarter of ring

This requires cross section at A does not rotate and is true from symmetry. Now consider increment of ring section defined by angle θ, and dθ. Bending moment at this section is given as $M = M_A - \frac{F(R-x)}{2}$, where, $x = R\cos\theta$, hence

$$M = M_A - \frac{FR(1-\cos\theta)}{2} \qquad \ldots(2)$$

Strain energy U is given as

$$U = \int \frac{(M^2 R d\theta)}{2EI} \qquad \ldots(3)$$

According to Eq. (1), partial derivative of U with respect to M_A must be zero. Therefore

$$\partial U / \partial M_A = \left(\frac{R}{EI}\right) \int_0^{\pi/2} M \left(\frac{\partial M}{\partial M_A}\right) d\theta \qquad \ldots(4)$$

Also, from Eq. (2), $\frac{\partial M}{\partial M_A} = 1$, and $\int_0^{\pi/2} M^2 d\theta = \int_0^{\pi/2} \left[M - \left(\frac{FR}{2}\right)(1-\cos\theta) \right] = 0$.

Integrating for given limits and solving for M_A ($\theta = \pi/2$), one gets $M_A = FR\left(\frac{1}{2} - \frac{1}{\pi}\right)$ = 0.182 *FR*. Solving for M by putting value of M_A in Eq. (2), one gets

$$M = \left(\frac{FR}{2}\right)\left(\cos\theta - \frac{2}{\pi}\right) \qquad \ldots(5)$$

Now, Castigliano's second theorem also states that, partial derivative U with respect to a load yields displacement component of loaded point in the direction of that load such as

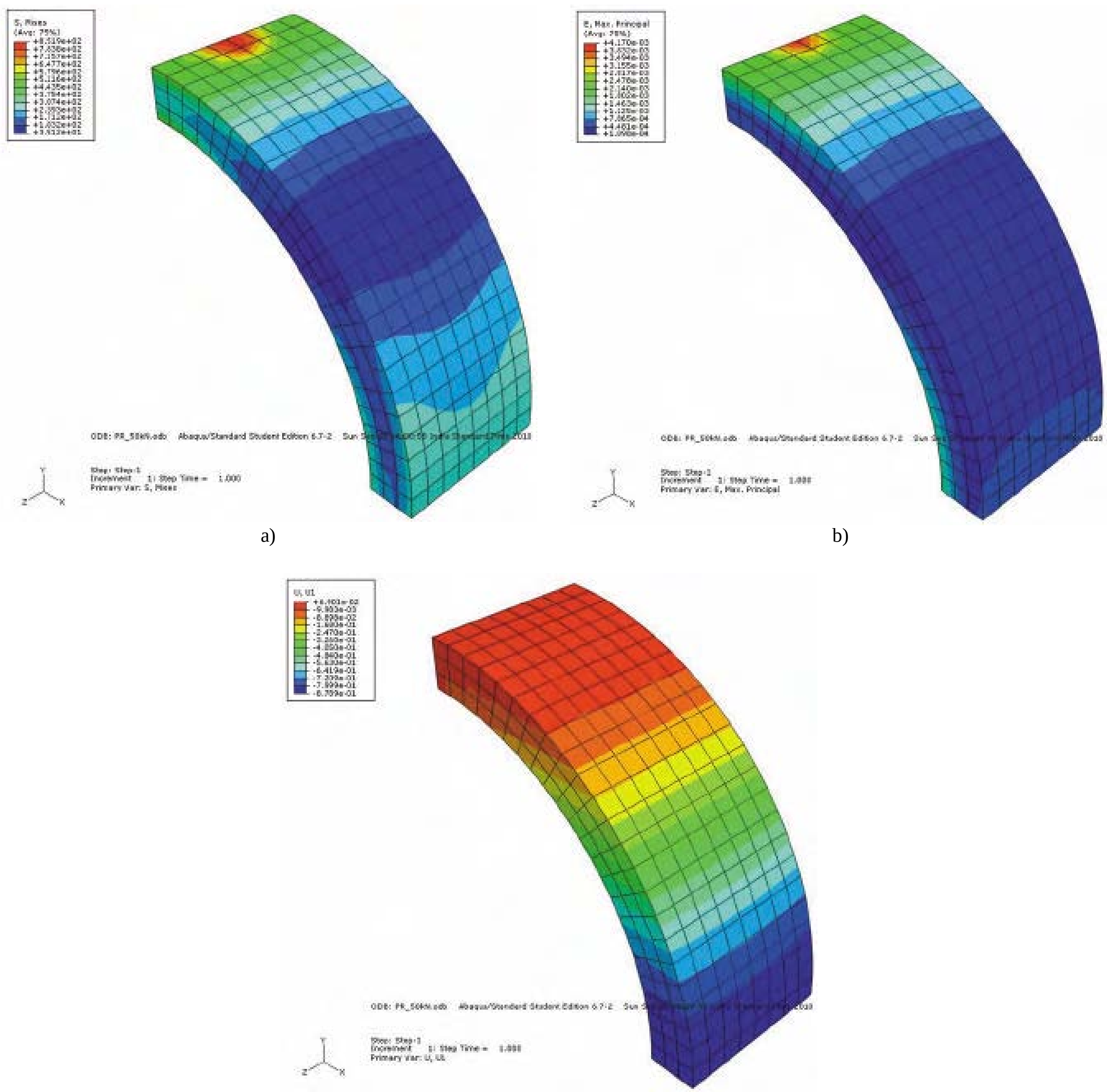
$$\delta_i = \frac{\partial U}{\partial F}, \ldots.. (i = 1, 2, \ldots n) \qquad \ldots(6)$$

Thus deflection of ring is obtained as

$$\delta = \delta_1 = \left(FR^3 / EI\right) \left(\pi/4 - 2/\pi\right) \qquad \ldots(7)$$

Axial deflection computed using Eq (7) is found consistent with reported[2,5-10] driven analytical expressions. Eq. (7) further suggests following relationship between different entities of a ring shaped FT: If capacity *F* and material *E* being constant for given FT and material, $\delta \propto \left(R^3/t^3\right)$ or $\delta \propto \left(R/t\right)^3$ $\delta \propto \left(1/b\right)$. Hence, for designing thin rings, (R/t) and b plays a vital role along with *F* and *E* of FT[5]. Higher the axial deflection under axial force applied, higher is the sensitivity of FT. Hence, axial deflection of FT should be large enough, within elastic limits of ring. Dimensions of two FTs (capacities, 20 kN & 50 kN) for axial forces in tension as well as compressions have been found on the basis of analytically driven expressions, considering modulus of elasticity (210 GPa) and Poission ratio (0.3).

Stresses have been calculated for FT using theory of bending of curved bars for rings of rectangular cross section[14] and are found to be in permissible stress range for material selected. Dimensions and other details of FT for two capacities are given as: 20 kN (inner radii, 86 mm; outer radii, 96 mm; thickness, 10 mm; width, 45 mm; Young's modulus, 210 GPa; Poission ratio, 0.3;



a) b) c)

Fig. 2 — Finite element analysis of ring shaped FT under axial applied in tension mode for: a) Stress distribution; b) Strain distribution; and c) Axial deflection

and axial deflection, 2.82 mm); and 50 kN (inner radii, 80 mm; outer radii, 95 mm; thickness, 15 mm; width, 50 mm; Young's modulus, 210 GPa; Poission ratio, 0.3; and axial deflection, 1.69 mm).

Finite Element Model and Analysis

Software ABAQUS standard student edition 6.7.2 has been used for modeling and analysis of FTs developed. A three dimensional quarter of idealized ring has been designed and suitable boundary conditions have been defined for finite element analysis (FEA). A three dimensional solid continuum 8 node element with reduced integration is considered and analysis is of linear type. Axial force is applied in tension mode as an indication of methodology using ABAQUS for studying stress - strain and axial deflection pattern. Suitable procedure for FEA

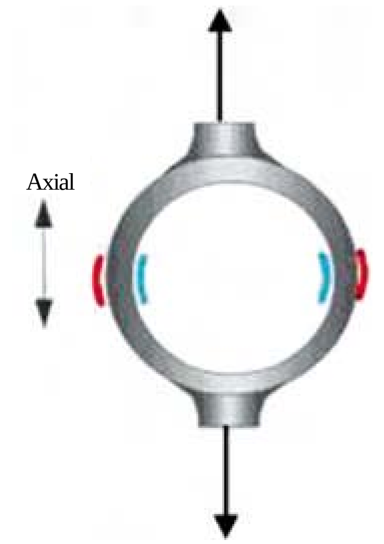


Fig. 3 — Arrangement of strain gauges over force transducer (FT)

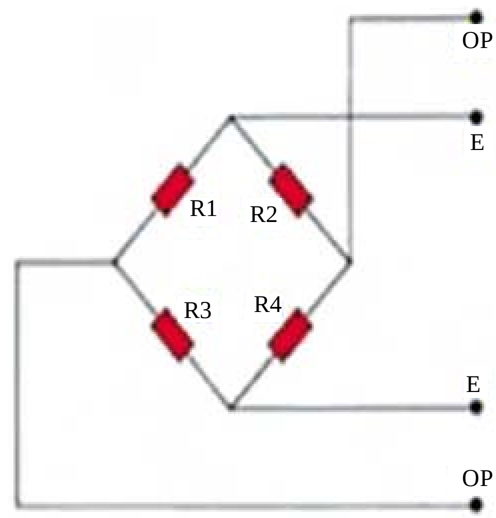


Fig. 4 — A typical Wheatstone bridge

of the quarter of ring has been adopted and stress, strain and axial deflection patterns have been evaluated. Results of FEA have been summarized in the form of stress, strain and deflection patterns[11] of ring shaped FT under axial applied in tension mode.

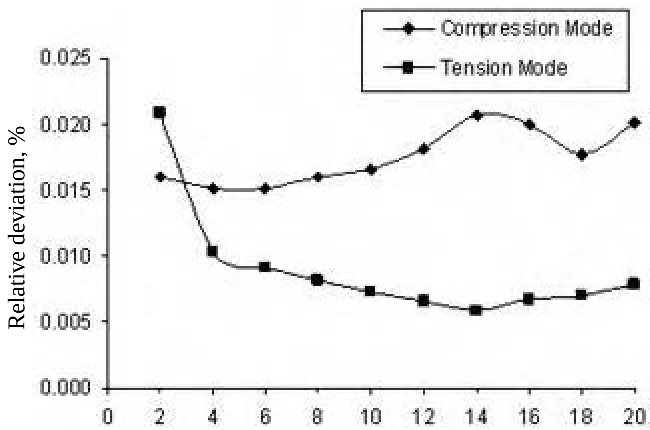
Maximum stress (Fig. 2a) occurs at upper top point where force is applied. From the point of application of axial force, along periphery, stress tends to decrease and at 39.6^o angle, stress is least and it continues to increase along periphery for rest of the quarter of ring. Strain distribution (Fig. 2b) follows pattern similar to stress distribution and is minimum at 39.6^o angle. Axial deflection (Fig. 2c) is highest at free end, but it tends to decrease along the periphery of quarter and is almost negligible at other end of quarter. One end of the quarter of ring is considered fixed and no rotation has been permitted, hence, axial deflection has been almost negligible there. According to stress / strain distribution of ring quarter, stress / strain are found moderate at 90^o and hence, may be suitable place for implanting strain gauges.

Strain Gauge Implantation

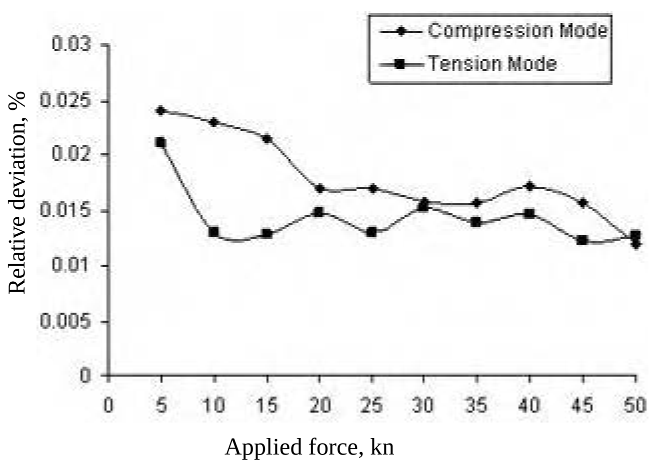
FTs (capacities 20 kN & 50 kN) for axial forces in tension mode and compression mode has been designed and studied using analytical derived expressions and FEA. Strain gauges have been mounted at an angle 90^o from vertical axis as explained by FEA that there stress / strain are sufficient enough. A balanced Wheatstone bridge has been constructed by mounting four strain gauges over FT. Two strain gauges have been mounted over outer surface, while rests are placed at inner surface (Fig. 3). Hence, strain is being measured by strain gauges instead of axial deflection by micrometer / vibrating reed or dial gauges in commonly used ring shaped FTs. Before strain gauges being implanted, machined elements are normalized by applying maximum axial force so that stress is in operational range. For fixing strain gauges, surface is flattened and surface roughness is restricted to few microns. Strain gauges are fixed using a hot curing adhesive from horizontal axis of sensing element. Proper curing and post curing of strain gauges has been done and connections are made as per Wheatstone bridge configuration to nullify temperature effect (Fig. 4). However, if any force is applied, Wheatstone bridge gets unbalanced and electrical output is generated in terms of mV/V, which reflects force applied.

Metrological Characterization

FTs developed have been metrologically characterized using 50 kN dead weight force machine (DWFM) (designed and developed by More House Corporation, USA as per the instructions of NPL, India), having uncertainty of force applied, cmc ± 0.003% (k = 2)[12], according to calibration procedure based on ISO 376-2004[13]. DWFM employs 18 stainless steel dead weights of nominal force values (0.5 - 5 kN), which take into account of local values of gravity and buoyancy correction for applied axial force by DWFM. Main components of DWFM are loading hanger, sets of dead weights and a rigid main frame supporting these



a)



b)

Fig. 5 — Uncertainty of measurement: a) 20 kN FT; and b) 50 kN FT

components. Pneumatic system has been used for loading and unloading weights to minimize oscillations so that force is stabilized in least possible time once applied or removed. Force (0.5 - 50 kN) may be applied by force and up to 50 KN forces may be applied in either mode (tension and compression) in any sequence desired depending upon the force to be applied. A high resolution digital indicator DMP40 (HBM Germany make) has been used for recording observations.

Calibration process is as follows: i) Digital indicator was switched on for 30 min to warm up and stabilized for no load output before start of calibration, no load output was noted (before taring) and calibration signal was noted; ii) Before application of calibration forces, FT was preloaded thrice to its maximum capacity and kept at full load for 90 s; iii) Calibration of FT has been done in tension mode as well as compression mode; iv) Calibration was carried out by applying two series of calibration forces in ascending order from 10% to 100% in steps of 10% at initial position, considered 0^o; v) Two series of calibration forces have been applied at rotation positions 120^o and 240^o; vi) FT was subjected to full load once for 90 s each time before starting calibration to new position; vii) Between loadings, readings corresponding to no load after waiting at least 30 s for return to zero were noted; and viii) Uncertainty of measurement of FT involves relative deviations due to zero error, repeatability error, reproducibility error, resolution error, interpolation error and uncertainty of measurement of force due to force machine. Metrological characterization has been summarized (Fig. 5).

Results and Discussion

FTs (20 kN & 50 kN) in tension and compression mode have been designed, developed using elastic theories of thin rings. FTs have been strain gauged and electrical circuit has been made according to Wheatstone bridge. As external force is applied, Wheatstone bridge is unbalanced and output is reflected in terms of mV/V, which is indication of force applied. FTs have been metrologically characterized using 50 kN [DWFM and calibration procedure based on ISO 376-2004. FTs have been found to exhibit good metrological results with uncertainty of measurement ± 0.025% (k = 2) for both modes i.e. tension and compression mode and confirms to class 00 as per ISO 376-2004.

Conclusions

FTs (20 kN & 50 kN) have been developed on the basis of theories used in practice. FTs developed have been metrologically studied using 50 KN DWFM and has been found to have good results. Uncertainty of FTs is found to be ± 0.025% (k = 2), which includes relative deviations due to zero error, repeatability error, reproducibility error, resolution error, interpolation error and uncertainty of measurement of force due to force machine itself. This study further provides scope for studying effect of nominal quantities like dimensions, Poission ratio etc. on the design of FTs.

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Symbols

b width of the cross section of ring, mm
t thickness of cross section of ring, mm
R_i inner radius of ring, mm
R_o outer radius of ring, mm
R mean radius (mm)
F applied force, N
E young's modulus of elasticity, GPa
U strain energy, J
M_A moment due to force at A, Nmm
M moment due to force at any position, Nmm
q angle of segment of ring, radian
d deflection of the ring, mm

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